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Ranking image segmentation methods using Data Envelopment Analysis

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Abstract

This study leverages a model based on Data Envelopment Analysis (DEA) to rank various image segmentation methods, identifying the most efficient approach for specific scenarios. By employing two case studies, we illustrate the application and effectiveness of DEA in comparing computational efficiency and segmentation accuracy across methods. The Chan-Vese method demonstrated notable performance, particularly at the regularization parameter $\mu=0.01$, where it achieved optimal efficiency in three cases. Similarly, the Bernard method, utilizing variational approaches with B-splines for contour representation, excelled at the smoothing parameter h=3, achieving the highest efficiency in three cases as well.

This research emphasizes the critical role of selecting suitable segmentation algorithms tailored to image characteristics, computational demands, and accuracy requirements. By incorporating metrics such as average computation times and Mean Sum of Square Distance (MSSD) accuracy, the study provides an objective framework for performance comparison. Results indicate significant variations in method efficiency based on input parameters, underscoring the importance of adaptive parameter tuning for achieving optimal outcomes. The findings contribute to advancing the evaluation and optimization of segmentation techniques, offering valuable insights for applications in medical imaging, geoinformatics, and beyond.

Keywords: Data Envelopmant Analysis, Ranking, AP, Super-Efficiency, image segmentation methods

I. INTRODUCTION

Image segmentation is a critical process in image analysis that involves partitioning an image into meaningful regions or objects.

This process has wide-ranging applications, including medical imaging, geoinformatics, multimedia processing, object recognition, and autonomous systems [1, 2, 3, 4] and etc. For instance, in the medical field, segmentation techniques are essential for identifying tumors, organ boundaries, or other critical regions in diagnostic images. In geoinformatics, segmentation aids in identifying geographical features, such as rivers, forests, or urban areas, from satellite imagery. The growing demand for accurate and efficient segmentation methods has motivated researchers to explore novel algorithms and evaluate their performances.

There are many image segmentation methods. It is important to find which one is more suitable and efficient for a specific class of images.

One of the key challenges in image segmentation is determining which method performs best for specific scenarios. Various factors, such as image characteristics, computational cost, and accuracy requirements, influence the effectiveness of segmentation algorithms.

For instance, methods like region-based approaches (e.g., Chan-Vese) and variational methods (e.g., Bernard's B-spline level set) have demonstrated varying degrees of success across different image classes, including animals, buildings, faces, and natural objects.

To address this evaluation challenge, Data Envelopment Analysis (DEA) has emerged as a powerful tool. DEA is a non-parametric, mathematical approach used to assess the relative efficiency of decision-making units (DMUs) based on input-output relationships. Originally developed for productivity analysis in economics and management, DEA has since been applied in diverse fields such as healthcare, engineering, and environmental studies (see [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,17, 18, 19, 20, 21, 22]). Its adaptability makes it well-suited for ranking image segmentation methods by considering both their computational efficiency (inputs) and segmentation accuracy (outputs).

We can also use DEA in ranking (see [23, 24, 25]). In this study, DEA is employed to rank various segmentation

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methods to identify the best-performing algorithm for specific image classes.

By utilizing metrics like average computation times and Mean Sum of Square Distance (MSSD) accuracies, this research aims to provide an objective comparison of widely-used segmentation approaches.

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II. MAIN METHOD

In this section, we explain two image segmentation methods and DEA model that we would like to use.

The study focuses on two prominent segmentation methods: Chan-Vese and Bernard's B-spline level set. These methods were chosen for their wide applicability and contrasting approaches to segmentation.

Image Segmentation Methods

Chan-Vese: The Chan-Vese method is a region-based segmentation approach that relies on minimizing an energy functional. This functional incorporates terms for the intensity of regions inside and outside the contour, as well as regularization terms to ensure smoothness. This method achieves the boundaries of an image by minimizing the following functional:

$$F(\mathrm{a}_1,\mathrm{a}_2,\emptyset)=\mu\int_{\mathrm{D}}|\nabla\left(\mathrm{H}(\emptyset(\mathrm{x}))\right)|\mathrm{dx}+\int_{\mathrm{D}}|\mathrm{l}(\mathrm{x})-\mathrm{a}_1|^2\mathrm{H}(\emptyset(\mathrm{x}))\mathrm{dx}+\int_{\mathrm{D}}|\mathrm{l}(\mathrm{x})-\mathrm{a}_2|^2\left(1-\mathrm{H}(\emptyset(\mathrm{x}))\right)\mathrm{dx} \quad (1)$$

in which $\emptyset$ is a signed distance function to the boundary, H is the Heaviside function, a_1 and a_2 are two constants that represent the mean pixel intensity inside and outside the $\emptyset$ (boundary) respectively and μ is the regularization parameter. By assuming that $\emptyset$ is fixed, we can obtain a_1 and a_2 as

$$a_1=\frac{\int_{\mathrm{D}}\mathrm{l}(\mathrm{x})\mathrm{H}(\emptyset(\mathrm{x}))\mathrm{dx}}{\int_{\mathrm{D}}\mathrm{H}(\emptyset(\mathrm{x}))\mathrm{dx}},\ a_2=\frac{\int_{\mathrm{D}}\mathrm{l}(\mathrm{x})\left(1-\mathrm{H}(\emptyset(\mathrm{x}))\right)\mathrm{dx}}{\int_{\mathrm{D}}\left(1-\mathrm{H}(\emptyset(\mathrm{x}))\right)\mathrm{dx}} \quad (2)$$

and Euler-Lagrange equation of Functional (1) is as following

$$\frac{\partial\emptyset}{\partial t}=\delta(\emptyset(x))(\mu\kappa-|l(x)-\mathrm{a}_1|^2+|l(x)-\mathrm{a}_2|^2) \quad (3)$$

The method iteratively updates the contour $\emptyset$ to minimize the functional, yielding an optimal segmentation.

By using (2) and (3), Chan-Vese method obtains boundaries. For more information about this method refer to [26].

Bernard: This method is considered a region-based method and is newer than the previous ones. This method uses a variational approach with B-splines to represent contours.

By expressing the contour as a linear combination of B-spline basis functions, it achieves a balance between accuracy and computational efficiency. In this method, the following functional is minimized:

$$\int_{\mathrm{D}}|\mathrm{l}(\mathrm{x})-\mathrm{b}_1|^2\mathrm{H}(\emptyset(\mathrm{x}))\mathrm{dx}+\int_{D}|\mathrm{l}(\mathrm{x})-b_2|^2\left(1-\mathrm{H}(\emptyset(\mathrm{x}))\right)\mathrm{dx} \quad (4)$$

we consider $\emptyset$ as a linear combination of B-spline basis functions:

$$\phi(x)=\sum_{k\in\mathrm{Z}^d}b[k]\beta^n(\frac{X}{h}-k) \quad (5)$$

in which $\beta^n(.)$ is the uniform symmetric n-degree B-spline. b[k] is the coefficients of B-spline representation and h is a parameter that decides the level of smoothness of contour (zero level set of $\emptyset$). For more information, see [27].

DEA

Data Envelopment Analysis is a non-parametric method that considers the relationship between inputs and outputs in decision-making units. This method is used to compare efficiency, productivity, etc

Data Envelopment Analysis, introduced by Charnes, Cooper, and Rhodes in 1978, is widely recognized as a benchmark for evaluating relative efficiencies in systems with multiple inputs and outputs.

Unlike parametric methods, DEA does not require predefined functional forms or distributions.

This flexibility has enabled its application in performance measurement across numerous domains. For example:

- Healthcare: Evaluating hospital efficiency by comparing resources (inputs such as staff and equipment) to patient outcomes (outputs like recovery rates).

- Environmental Studies: Assessing the sustainability of industries by comparing energy consumption and waste generation to production levels.
- Engineering: Comparing productivity in manufacturing plants or energy sectors.

In the context of image segmentation, DEA's ability to integrate diverse performance metrics makes it an ideal choice for algorithm evaluation.

By defining segmentation methods as DMUs and incorporating computational times as inputs and segmentation accuracy as outputs, DEA enables a holistic comparison of these techniques.

In this study, our decision making units (DMUs) are the image segmentation methods that we want to compare with each other. For each decision-making unit, we define inputs and outputs appropriate to the intended purpose.

In this study, we use the super-efficiency model for ranking (AP) [28]. Given that the nature of the model is input-oriented, the more efficiency score of the decision-making unit has the more efficient performance.

Suppose we have n decision-making units, with m inputs and s outputs. The model used has the property of constant returns to scale. The ranking model considered is as follows:

Efficiency: φ
Input: x_{ij}
Output: y_{rj}
Weighting factor: λ_j

$$
\begin{aligned}
&(\text{AP Score}):\quad \text{Min } \varphi \\
&\sum_{j=1\ \&\ j\neq o}^{n} \lambda_j x_{ij} \leq \varphi x_{io} && \forall i=1,2,\ldots,m \\
&\sum_{j=1\ \&\ j\neq o}^{n} \lambda_j y_{rj} \geq y_{ro} && \forall r=1,2,\ldots,s \\
&\lambda_j \geq 0 && \forall j=1,2,\ldots,n
\end{aligned}
\tag{6}
$$

Table 1. Average computation times for categories(Input).

Method/Category	Animals	Buildings	Faces	Nature	Objects
Chan-Vese with μ = 0.01	42.1287s	55.9337s	47.6588s	41.7300s	56.6916s
Chan-Vese with μ = 0.2	42.3996s	52.8610s	50.5395s	39.1866s	64.7076s
Chan-Vese with μ = 0.4	45.0279s	52.7354s	53.1786s	42.2463s	66.4899s
Chan-Vese with μ = 0.6	44.6564s	51.2075s	64.2735s	44.3152s	67.6379s
Chan-Vese with μ = 0.8	48.3813s	48.6384s	62.2584s	49.1314s	67.5419s
Chan-Vese with μ = 1	53.6698s	52.1388s	54.9031s	51.8473s	67.5723s

Table 2. Average accuracy (MSSD) for every categories (Output).

Method/Category	Animals	Buildings	Faces	Nature	Objects
Chan-Vese with μ = 0.01	1462	1972	1723	2527	2625
Chan-Vese with μ = 0.2	1401	2195	1757	4235	2246
Chan-Vese with μ = 0.4	1587	2319	1855	4765	2332
Chan-Vese with μ = 0.6	1881	2743	1889	4899	2344
Chan-Vese with μ = 0.8	2175	3766	1910	4922	2407
Chan-Vese with μ = 1	2237	3913	1849	5162	2787

III. CASE STUDY

The DEA model applied in this study provides a detailed ranking of image segmentation methods based on their efficiency scores.

For ranking segmentation methods, the study employs a super-efficiency DEA model. This model evaluates each DMU (i.e., segmentation method) based on its relative efficiency, considering:

- Inputs: Average computation times for different image categories (e.g., animals, buildings).
- Outputs: Accuracy measures (e.g., inverse MSSD).

The computation time that an image segmentation algorithm consumes is considered as an input. The inverse of Mean Sum of Square Distance (MSSD) is considered as an output.

We use data from the paper by Bozorgmanesh (2021) in which he examined the Chan-Vese model for 6 cases [17]. The Table 1 and 2 show the inputs and outputs.

The results that we obtain using the ranking model is in Table 3.

According to the results obtained, Chan-Vese method works efficiently in three cases, but the best one is for the $\mu = 0.01$. In general, it is clear that the best case is for $\mu = 0.01$ and the worst case is for $\mu = 1$.

Also, in the mentioned paper, he examined the Bernard model for 4 cases, whose inputs and outputs are in Table 4 and 5. According to the results obtained, Bernard method works efficiently in three cases, but the best one is for the $h = 3$. In general, it is clear that the best case is for $h = 3$ and the worst case is for $h = 1$.

In the other word, the results indicate significant variations in the performance of these methods across different image categories. For example:

Chan-Vese Method:

The performance of the Chan-Vese method varies based on the regularization parameter μ. The analysis reveals that smaller values of μ (e.g., 0.01) result in higher efficiency scores due to lower computational times and better segmentation accuracies for certain classes (e.g., animals, faces).

Conversely, higher μ values increase computational times and degrade segmentation accuracy, particularly for complex image classes like objects.

Bernard's B-spline Level Set:

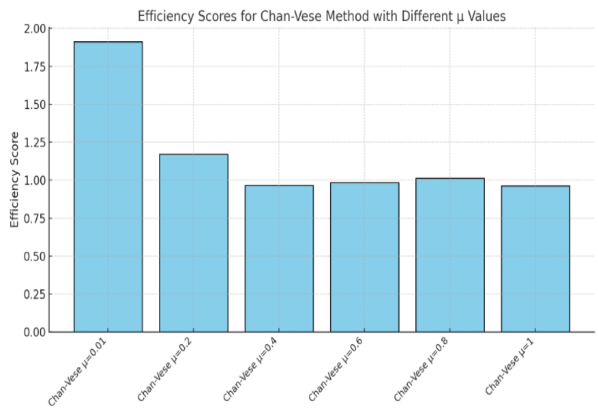
The efficiency of this method improves with higher smoothing parameters (e.g., h=3), striking a balance between segmentation accuracy and computational efficiency.

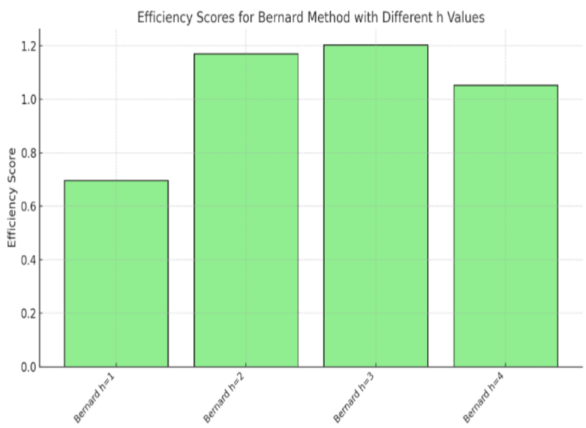
For lower smoothing parameters (e.g., h=1), the method demonstrates reduced efficiency, likely due to over-segmentation and increased computational costs.

A comparison of the two methods highlights distinct strengths and weaknesses:

- The Chan-Vese method is better suited for images with distinct intensity contrasts, making it highly efficient for simpler categories like animals and faces.
- Bernard's method, with its B-spline representation, excels in handling high-resolution images with intricate boundaries, such as buildings and objects.

For bertter understanding, we use some plot as follow:





IV. CONCLUSION AND FUTURE WORK

This study demonstrates the utility of DEA in evaluating and ranking image segmentation methods. By integrating computational times and segmentation accuracies as performance metrics, the DEA model provides a comprehensive framework for algorithm assessment. Key findings include:

1. The Chan-Vese method performs optimally for specific regularization parameters (e.g., μ=0.01) and simpler image classes.

2. Bernard's method achieves superior results for high-resolution and complex images with appropriate smoothing parameters (e.g., h=3).

Things we plan to do in the future include:

- Explore additional segmentation methods and their integration into the DEA framework.
- Investigate the impact of different DEA models (e.g., variable returns to scale) on ranking outcomes.
- Extend the analysis to dynamic images and real-time segmentation scenarios.

Table 3. Result (AP Score)

Method	AP Score	Rank
Chan-Vese with μ = 0.01	1.913	1
Chan-Vese with μ = 0.2	1.171	2
Chan-Vese with μ = 0.4	0.965	5
Chan-Vese with μ = 0.6	0.984	4
Chan-Vese with μ = 0.8	1.014	3
Chan-Vese with μ = 1	0.963	6

Table 4. Average computation times for categories(Input).

Method/Category	Animals	Buildings	Faces	Nature	Objects
Bernard with h = 1	95.7417s	122.1563s	133.6571s	101.3189s	122.8833s
Bernard with h = 2	52.6740s	49.6221s	48.3326s	48.6027s	46.8617s
Bernard with h = 3	41.4365s	38.0190s	38.7606s	37.3402s	39.0221s
Bernard with h = 4	43.1294s	38.8083s	39.2178s	38.3552s	38.2661s

Table 5. Average accuracy (MSSD) for every category (Output).

Method/Category	Animals	Buildings	Faces	Nature	Objects
Bernard with h = 1	1121	1765	1664	2015	2524
Bernard with h = 2	1003	1739	1601	2766	3615
Bernard with h = 3	1212	2445	1798	2936	4065
Bernard with h = 4	1421	3520	2656	3092	3935

Table 6. Result (AP Score)

Method	AP Score
Bernard with h = 1	0.697
Bernard with h = 2	1.171
Bernard with h = 3	1.204
Bernard with h = 4	1.053

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